LEVEL OF CREEP SENSITIVITY IN COMPOSITE STEEL-CONCRETE BEAMS ACCORDING TO ACI 209R-92 MODEL, COMPARISON WITH EUROCODE-4 (CEB MC90-99)

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The paper presents analysis of the stress and deflections changes due to creep in statically determinate composite steel-concrete beam. The mathematical model involves the equation of equilibrium, compatibility and constitutive relationship, i.e. an elastic law for the steel part and an integral-type creep law of Boltzmann-Volterra for the concrete part. On the basis of the theory of the viscoelastic body of Arutyunian-Trost-Bažant for determining the redistribution of stresses in beam section between concrete plate and steel beam with respect to time 't', two independent Volterra integral equations of the second kind have been derived. Numerical method based on linear approximation of the singular kernal function in the integral equation is presented. Example with the model proposed is investigated. The creep functions is suggested by the ACI 209R-92 model. The elastic modulus of concrete $E_c(t)$ is assumed to be constant in time 't'. The obtained results are compared with the results from the model CEB MC90-99.

Keywords : *composite steel-concrete section, Volterra integral equations, rheology, linear approximation, ACI209R-92, EUROCODE-4, singular kernal function*

1. Introduction

Steel-concrete composite beams are wide spread form of construction in both buildings and bridges.

The time-varying behavior of composite steel-concrete members under sustained service loads drawn the attention of engineers who were dealing with the problems of their design more than 60 years [56].

The solution of structural problems involving creep and shrinkage phenomena in composite steel-concrete beams has been an important task for engineers since the first formulation of the mathematical model of linear viscoelasticity. If on one hand the definition of a suitable formulation of creep laws involved scientists and researchers in past decades and many prediction models have been developed, starting from experimental data and from the direct observation of the long term behavior of concrete structures (Branson & Christiason [38], Müller [70], Bažant & Baweja 2000 [12, 13], Gardner & Lockman 2001 [57]), the development of structural analysis procedures, based on the creep models, is on the other hand, of great interest for engineers who need to investigate the effects of creep and shrinkage on the structures they design.

Creep and shrinkage have a considerable impact upon the performance of composite beams, causing increased deflection as well as affecting stress distribution. Creep in concrete

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represents dimensional change in the material under the influence of sustained loading. Failure to include creep and shrinkage effects in the analysis of the composite steel-concrete beams may lead to excessive deformation and caused significant redistribution of stress between concrete plate and steel beam.

In general, time-dependent deformation of concrete may severely affect the serviceability, durability and stability of structures (Chiorino, M., Sassone, M., Bigaran, D., Casalegno, C. [43].

The first works, which give the answer to this problem are based on the Law of Dischinger [49, 50] (theory of aging), who had first formulated a time-dependent stress-strain differential relationship for concrete, using the following equation:

$$\frac{\mathrm{d}\varepsilon_{\mathrm{ct}}}{\mathrm{d}t} = \frac{\sigma_{\mathrm{ct}}}{E_{\mathrm{c0}}}\frac{\mathrm{d}\varphi_t}{\mathrm{d}t} + \frac{1}{E_{\mathrm{ct}}}\frac{\mathrm{d}\sigma_{\mathrm{ct}}}{\mathrm{d}t}\ , \tag{1}$$

where φ_t is called creep function.

These books and papers connected with the names of Fröhlich [56], Esslinger [54], Klöppel [64], Sonntag [94], Kunert [67], Dimitrov [48], Mrazik [69] and Bujňák [39] represent one independent group for which it is characteristic that by writing equilibrium and compatibility equations and the constitutive laws for the two materials, the problem is governed by a system of two simultaneous differential equations, which have been derived and solved.

As known in this differential equations it exists a group of normal forces $N_{\mathrm{c,r}}(t)$, $N_{\mathrm{a,r}}(t)$ and bending moments $M_{\mathrm{c,r}}(t)$, $M_{\mathrm{a,r}}(t)$, which influence the general stress conditions of the statically determinate composite plate beam is expressed by the decrease of the stresses in the concrete plate and in the increase of stresses in the steel beam (Fig. 1).

All these methods have been collected and analyzed by Sattler [88] and by the first author of this paper [72].

In parallel with the developed analytical methods, Blaszkowiak [36], Bradford [37], Fritz [55] and Wippel [101] have developed approximate methods, which use Dischinger's idea for applying in the calculation the ideal (fictitious) modulus of elasticity [49, 50]:

$$E_{\mathrm{ci}} = \frac{E_{\mathrm{c0}}}{1+\varphi_{\mathrm{n}}}\ , \tag{2}$$

where φ_{n} is the ultimate value of creep.

Another method of the estimate design calculation as described in Schrader [91] has been based on the creep fibred method by Busemann [40].

With Wippel's methods [101] the first stage of the development of the analytical methods is based entirely on the works of Dischinger [49, 50], has been completed.

Further development of rheology as a fundamental science and its application to concrete [3, 5, 81, 86, 98] as well as a great number of investigations in the field of creep of concrete have led to new formulations of the time-dependent behavior of concrete [22, 41, 42, 61, 80].

These new formulations give the relationship between $\sigma_{\mathrm{c}}(t)$ and $\varepsilon_{\mathrm{c}}(t)$ are formulated by integral equations, which present the basis of the theory of linear viscoelastic bodies.

The integral-type creep law, i.e., the superposition equation for uniaxial prescribed stress history $\sigma(t)$, is expressed by:

$$\varepsilon_{\mathrm{c}}(t,t_0) = \varepsilon^{\mathrm{sh}}(t) + \sigma(t_0)\,J(t,t_0) + \int_{t_0}^{t} \frac{\mathrm{d}\sigma(t)}{\mathrm{d}\tau}\,J(t,\tau)\,\mathrm{d}\tau\ . \tag{3}$$

By using algebraic methods, simpler forms for (3) are obtained. These methods are based on the hypothesis that the strain in the concrete fibers can be considered as a linear function of the creep coefficient (Trost [96], Bažant [8]). This permits transforming (3) into

$$\begin{aligned}\varepsilon_{\mathrm{c}}(t,t_0) = \varepsilon^{\mathrm{sh}}(t) + \sigma_{\mathrm{c}}(t_0)\left[\frac{1}{E_{\mathrm{c}}(t_0)} + \frac{\varphi(t,t_0)}{E_{\mathrm{c}}}\right] + \\ + [\sigma_{\mathrm{c}}(t) - \sigma_{\mathrm{c}}(t_0)]\left[\frac{1}{E_{\mathrm{c}}(t_0)} + \frac{\chi(t,t_0)\,\varphi(t,t_0)}{E_{\mathrm{c}}}\right]\ ,\end{aligned} \tag{4}$$

where

$$\chi(t,t_0) = \frac{E_{\mathrm{c}}(t_0)}{E_{\mathrm{c}}(t_0) - R(t,t_0)} - \frac{E_{\mathrm{c}}}{E_{\mathrm{c}}(t_0)\,\varphi(t,t_0)} \tag{5}$$

is the aging coefficient; $\varphi(t,t_0)$ – the creep coefficient; $R(t,t_0)$ – relaxation function, i.e., the stress response to a constant unit strain applied at the time t_0; E_{c} – the elastic modulus of concrete at 28 days.

The age-adjusted effective method (AAEM) directly assumed the expression provided by (5) for the aging coefficient. In this case, it is necessary to evaluate previously the relaxation function $R(t,t_0)$. This function is calculated numerically by applying the step-by-step procedure of the general method to the integral type relation between the creep and the relaxation function Bažant [7]. However, for some standard parameters, diagrams of the χ coefficient are available from model codes Chiorino [41]). Moreover, a number of empiric expressions were recently proposed that provide final values of the χ coefficient with sufficient precision.

Using the effective modulus method (EMM), (4) becomes

$$\varepsilon_{\mathrm{c}}(t,t_0) = \varepsilon^{\mathrm{sh}}(t) + \sigma(t_0)\,J(t,t_0)\ , \tag{6}$$

where $\chi(t,t_0) = 1$ and $E_{\mathrm{c}}(t_0) = E_{\mathrm{c}}$. In this case, the variation of the stress in the interval $(t - t_0)$ is neglected and the stress is always considered equal to its final value. Consequently, this method underestimates the creep effects when the stress decreases with time. The time dependent analysis can be performed as an equivalent elastic analysis, where Young's modulus E_{c} is multiplied by the coefficient $1/[1+\varphi(t,t_0)]$.

When the Mean Stress Method (MSM) is applied (4) can be written as

$$\varepsilon_{\mathrm{c}}(t,t_0) = \varepsilon^{\mathrm{sh}}(t) + \sigma(t_0)\,J(t,t_0) + [\sigma_{\mathrm{c}}(t) - \sigma_{\mathrm{c}}(t_0)]\,\frac{J(t,t) + J(t,t_0)}{2}\ , \tag{7}$$

where $\chi(t,t_0) = 0.5$ and $E_{\mathrm{c}}(t_0) = E_{\mathrm{c}}$.

Equations (4), (6) and (7) represent the essence of the algebraic methods. It needs to be pointed out, however, that these algebraic equations used in structural analysis as constitutive laws for concrete in substitution of the integral-type creep law, as presented still cannot give realistic pictures of the stresses and deflections.

However, in order to avoid the mathematical problems in solving of the integral equations of Volterra for treating the problem connected with the creep of concrete structures, Trost [96] and Zerna [99], have revised the integral relationship into new algebraic stress-strain relationship:

$$\varepsilon_{\mathrm{c}t} = \frac{\sigma_{\mathrm{c}0}}{E_{\mathrm{c}0}}\,[1+\varphi_t] + \frac{\sigma_{\mathrm{c}t} - \sigma_{\mathrm{c}0}}{E_{\mathrm{c}0}}\,[1+\varrho\,\varphi_t]\ , \tag{8}$$

where ϱ is the relaxation coefficient. From the same considerations another revision of integral relationship into new algebraic stress-strain relationship has been made by Krüger [65] and Wolff [103]:

$$E_{c0}\,\varepsilon_{c\varphi,t} = \sigma_{c0}\,\frac{\varphi_{t0}-\varphi_{t1}}{2} + \sigma_{ct}\left[1+\frac{\varphi_{t(t-1)}}{2}\right] + \sum_{i=1}^{t-1}\sigma_{c,i}\,\frac{\varphi_{t,i-1}-\varphi_{t,i+1}}{2}\ . \tag{9}$$

On the basis of that algebraic stress-strain relationship, new methods have been developed connected with the names Wappenhans [100], Wolff [103], Trost [97], Heim [62], Amadio [2], Dezi [45–47, 95] (by preposition that the connectors are deformationsable) and Gilbert [58, 59], for solving the problem raised by Fröhlich [56].

In parallel with the methods developed by Kindman [63], Lapos [68], Pachla [71], Partov [75], on the basis of the theory of linear viscoelastic bodies, Sattler [89], Haenzel [60], and Profanter [79] have recently developed new methods, which are based on the 'modified theory' of Dischinger, called also the theory of Rüsch-Jungwirt [85]. This theory is described by the following equations:

$$\frac{d\varepsilon_{ct}}{dt} = \frac{\sigma_{ct}}{E_{cv}}\,\frac{d\varphi_{f,v}}{dt} + \frac{1}{E_{cv}}\,\frac{d\sigma_{ct}}{dt}\ , \tag{10}$$

where

$$E_{cv} = \frac{E_c(t_0)}{1.4}\ , \qquad \varphi_{f,v} = \frac{\varphi_{f,0}\,[K_f(t)-K_f(t_0)]}{1.4}\ .$$

Different approach to the solving of the formulated problems is applying the FEM by, Cumbo [44], Sassone [87] and Wissman [102].

Since the theory of Rüsch-Jungwirt [85] has been subjected to serious criticism in the works of Alexandrovski-Arutyunyan [3, 52, 93] and [6–21, 23, 24, 27–35] the authors of the present paper make an attempt for a new step toward deriving more precise solution of the problem. An effort is made to give an answer to the dispute between Bažant and Rüsch-Jungwirt in [25, 26].

The first works [73–76], which give the answer to this dispute [25, 26], using the integral equation of Volterra, are based on the Law of by Bolztmann-Volterra [3, 21, 93] who first formulated a time-dependent stress-strain differential relationship for concrete, described by the following integral equation:

$$\varepsilon_c(t) = \frac{\sigma_c(t_0)}{E_c(t_0)}\,[1+\phi(t-t_0)] + \int_{t_0}^{t}\frac{d\sigma_c(\tau)}{d\tau}\,\frac{1}{E_c(\tau)}\,[1+\phi(t-\tau)]\,d\tau\ , \tag{11}$$

where $\phi(t-\tau) = \varphi_N\,K(\tau)\,f(t-\tau)$ is the so called the creep function and φ_N the ultimate value of creep coefficient, $K(\tau)$ depends on the age increase of concrete. It is called the function of aging, and it characterizes the process of the aging. The increase of τ makes $K(\tau)$ monotonously decrease. The functions

$$K(\tau) = \begin{cases} \dfrac{10.28}{5+\sqrt{\tau}} & \tau \le 857 \\ 0.3 & \tau > 857 \end{cases} \qquad \text{and} \qquad f(t-\tau) = 1 - e^{\left[-0.6\left(\frac{t-\tau}{30}+0.0025\right)^{0.4}-0.091\right]}\ , \tag{12}$$

(where t is the time interval during which the structure is under observation, τ is the running coordinate of time) – characterizes the process of creeping.

A practical method for solving of composite constructions based on Volterra integral equations are reported in [73]. A new idea for development of the above mentioned method is the investigation of the tangent modulus of concrete elasticity besides invariant in time t i.e. $E_c(\tau) = E_c(t_0) = E_{const}$ and also for the case when it depends on time t [38, 92]:

$$E_c(\tau) = E_c(t_0) \sqrt{\frac{\tau}{4 + 0.86\,\tau}} \; . \tag{13}$$

A practical example with time-dependent elasticity modulus of concrete is considered in [77].

Since the new norms suggested by EUROCODE-4 [52, 53] in analysis of composite steel-concrete beams regarding rheology, required a new 'CEB-FIP' creep models code 1990, which leads to completely different approach for solving of the above formulated problems in was made attempt [78] to reformulate and solve these problems taking into account the new mathematical formulas in CEB MC90.

The CEB MC90 (Müller and Hilsdorf 1990, [70]) is intended to predict the time-dependent mean cross-section behavior of a concrete member. It has concept similar to ACI209R-92 model in the sense that it gives a hyperbolic change with time for creep and shrinkage, and also uses an ultimate value corrected according mixture proportioning and environmental conditions. The models are valid for normal weight plain structural concrete having an average compressive strength in the range of $20\,\mathrm{MPa} \le f_{cm28} \le 90\,\mathrm{MPa}$. The age of loading t_0 should be at least 1 day, and the sustained stress should not exceed 40 % of the mean concrete strength f_{cmt0} at the time of loading t_0. The CEB model does not require any information regarding the duration of curing and curing condition, but takes into account the average relative humidity and member size. Required parameters are: age of concrete when drying starts, usually taken as the age at the end of moist curing (day); age of concrete at loading (days);concrete mean compressive strength at 28 days (MPa); relative humidity expressed as a decimal; volume-surface ratio or effective cross-section thickness of a member (mm) and cement type.

The creep (compliance) function proposed by the 1990 CEB Model Code ('CEB-FIP' 1991) defined the strain at time t caused by a constant stress acting from time τ to time t, is given by relationsship

$$J(t, t_0) = \frac{1}{E_{cmt0}(t_0)} + \frac{\phi_{28}(t, t_0)}{E_{cm28}} \; , \tag{14}$$

where $\phi(t, t_0)$ gives the ratio of the creep strain since the start of loading at the age t_0 to the elastic strain due to a constant stress applied at a concrete age of 28 days; E_{cmt0} is the modulus of elasticity of concrete at the time of loading t_0 and E_{cm28} is the mean modulus of elasticity concrete at 28 days (MPa). Hence $1/E_{cmt0}$ represents the initial strain per unit stress at loading.

The creep coefficient is evaluated with following formula:

$$\phi(t, t_0) = \phi_0\, \beta_c(t - t_0) \; ,$$

where

$$\phi_0 = \phi_{RH}\, \beta(f_{cm})\, \beta(t_0)$$

or

$$\phi(t, t_0) = \phi_{RH}\, \beta(f_{cm})\, \beta(t_0)\, \beta_c(t - t_0) \; , \quad \phi(t, \tau) = \phi_{RH}\, \beta(f_{cm})\, \beta(\tau)\, \beta_c(t - \tau) \; ,$$

where

$$\phi_{RH} = 1 + \frac{1 - \dfrac{RH}{100}}{0.46\sqrt[3]{\dfrac{h_0}{100}}}$$

is a factor to allow for the effect of relative humidity on the notional creep coefficient. RH is the relative humidity of the ambient environment in %.

$$\beta(f_{\mathrm{cm}}) = \frac{5.3}{\left(\dfrac{f_{\mathrm{cm}}}{10}\right)^{0.5}}$$

is a factor to allow for the effect of concrete strength on the notional creep coefficient.

$$\beta(t_0) = \frac{1}{0.1 + t_0^{0.2}}$$

is a factor to allow for the effect of concrete age at loading on the notional creep coefficient (for continuous process we consider the function).

$$\beta(\tau) = \frac{1}{0.1 + \tau^{0.2}}$$

is a function of aging, depending on the age of concrete and it characterizes the process of aging.

$$\beta_{\mathrm{c}}(t - t_0) = \left[\frac{t - t_0}{\beta_{\mathrm{H}} + (t - t_0)}\right]^{0.3}$$

is a function to describe the development of creep with time after loading.

$$\beta_{\mathrm{H}} = 150\left[1 + \left(1.2\,\frac{RH}{100}\right)^{18}\right]\frac{h_0}{100} + 250 \le 1500$$

is a coefficient depending on the relative humidity (RH in %) and notional member size (h_0 in mm). $f_{\mathrm{cm28}} = f_{\mathrm{ck}} + 8$ if the mean compressive strength of concrete at the age of 28 days (megapascals) and $h_0 = 2\,A_{\mathrm{c}}/u$ the notional size of member (millimeters) (A_{c} – the cross section; and u – the perimeter of member in contact with the atmosphere); is the specified characteristic compressive cylindrical strength (MPa) below which 5 % of all possible strength measurement for the specified concrete may by expected to fall.

Constant Young's modulus is given by:

$$E_{\mathrm{cm28}} = 10^4\,(f_{\mathrm{cm28}})^{\frac{1}{3}}\ .$$

Variable Young's modulus is given by:

$$E_{\mathrm{cm}}(t) = \beta_{\mathrm{cc}}^{0.5}\,E_{\mathrm{cm28}}\ ,$$

where

$$E_{\mathrm{cm28}} = 10^4\,(f_{\mathrm{cm28}})^{\frac{1}{3}} \qquad \text{and} \qquad \beta_{\mathrm{cc}} = \exp\left[s\left(1 - \frac{5.3}{t^{0.5}}\right)\right]\ ,$$

where $s = 0.25$ for normal and rapid hardening cements. So

$$E_{\mathrm{cm}}(t) = 336190\,\mathrm{e}^{0.5\left[0.25\left(1-\frac{5.3}{\sqrt{t}}\right)\right]}$$

is a final creep coefficient of concrete.

In this paper we try to solve this problem according the mathematical model developed by (Branson and Christiason) [38], incorporated in developed model in ACI Committee 209R-92 and the results obtained comparing with this obtained from model CEB90-99 [78].

2. Basic equations for determining the creep coefficient according ACI 209R-92

This is an empirical model developed by (Branson and Christiason) [38] 1971, with minor modification introduced in ACI 209R-92. The models for predicting creep and shrinkage strains as a function of time have the same principle: a hyperbolic curve that tends to an asymptotic value called the ultimate value. The form of these equations is thought to be convenient for design purpose in which the concept of the ultimate (in time) value is modified by the time-ratio (time-dependent development) to yield the desired results. The shape of the curve and ultimate value depend on several factors such as curing conditions, age of application of load, mixture proportioning, ambient temperature and humidity.

The design approach presented for predicting creep refers to standard condition and correction factors for other than – standard condition. The corrections factors are applied to ultimate values. Because creep equation for any period is linear function of the ultimate values, however, the correction factors in this procedure may be applied to short-term creep.

Required parameters are: age of concrete when drying starts, usually taken as the age at the end of moist curing (day); age of concrete at loading (days); curing methods; ambient relative humidity expressed as a decimal; volume-surface ratio or average cross-section thickness of a member (mm); cement type; concrete slump in mm; fine aggregate percentage (%); cement content (kg/m^3) and air content of concrete expressed in percent (%). The last four parameters are not included in CEB MC90 model.

The creep (compliance) function proposed by the ACI 209R-92 model [1], that presents the total stress-dependent strain by unit stress is given by the relationship:

$$J(t,t_0) = \frac{1}{E_{\mathrm{cm}t0}} + \frac{\phi(t,t_0)}{E_{\mathrm{cm}t0}} = \frac{1+\phi(t,t_0)}{E_{\mathrm{cm}t0}}\ , \tag{15}$$

where $\phi(t,t_0)$ is the creep coefficient as the ratio of the creep strain to the elastic strain at the start of loading at the age t_0 (days) and $E_{\mathrm{cm}t0}$ is the modulus of elasticity at the time of loading t_0 (MPa), respectively.

The creep model proposed by ACI 209R-92 has two components that determine the ultimate asymptotic value and the time development of creep. The predicted parameter is not creep strain, but creep coefficient $\phi(t,t_0)$, (defined as the ratio of the creep strain to the initial elastic strain).

The creep coefficient is evaluated with the following formula:

$$\phi(t,t_0) = \phi_{\mathrm{u}}\,\beta_{\mathrm{c}}(t-t_0)\ ,$$

where $\phi(t,t_0)$ is the creep coefficient at the concrete age t_0 due to a load applied at the age t_0; $(t-t_0)$ is the time since application of load; ϕ_{u} is the ultimate creep coefficient.

For the standard conditions in the absence of specific creep date for local aggregates and conditions, the average value proposed for the ultimate creep coefficient ϕ_u is equal to 2.35.

For conditions other than standard conditions the value of the ultimate creep coefficient $\phi_u = 2.35$ needs to be modified by six correction factors, depending on particular conditions

$$\phi_u = 2.35\,\gamma_c \ ,$$

where:

$$\gamma_c = \gamma_{c,t0}\,\gamma_{c,RH}\,\gamma_{c,vs}\,\gamma_{c,s}\,\gamma_{c,\psi}\,\gamma_{c,\alpha}$$

and: $\gamma_{c,t0} = 1.25\,t_0^{-0.118}$ corresponds to $\beta(t_0)$ in CEB MC90, is a function of aging, depending on the age of concrete and it characterizes the process of aging;
$\gamma_{c,RH} = 1.27 - 0.67\,h$ for $h \geq 0.40$ is the ambient humidity factor, where the relative humidity h is in decimal – corresponds to ϕ_{RH} CEB MC90;

$$\gamma_{c,vs} = \frac{2}{3}\left(1 + 1.13\,\mathrm{e}^{-0.0213(V/S)}\right)$$

corresponds to β_H in CEB MC90), where V is the specimen volume in mm^3 and S the specimen surface area in mm^2, allows to consider the size of member in terms of the volume-surface ratio;
$\gamma_{c,s} = 0.82 + 0.00624\,s$ is slump factor, where $s = 75\,\mathrm{mm}$ is the slump of fresh concrete;
$\gamma_{c,\psi} = 0.88 + 0.0024\,\psi$ is fine aggregate factor, where $\psi = 40$ is the ratio of fine aggregate to total aggregate by weight expressed as percentage;
$\gamma_{c,\alpha} = 0.46 + 0.09\,\alpha \geq 1$ is air content factor, where $\alpha = 2$ is the air content in percentage.

$$\beta_c(t - t_0) = \frac{(t - t_0)^{0.6}}{10 + (t - t_0)^{0.6}}$$

is a function to describe the development of creep with time after loading.

The secant modulus of elasticity of concrete E_{cmt0} at any time t_0 of loading is given by $E_{cmt0} = 0.043\,\varrho_c^{1.5}\sqrt{f_{cmt0}}\,\mathrm{MPa}$, where ϱ_c is the unit weight of concrete ($\mathrm{kg/m^3}$) and f_{cmt0} is the mean concrete compressive strength at the time of loading (MPa). The general equation for predicting compressive strength at an time t is given by $f_{cmt} = f_{cm28}\,t/(a + b\,t)$, where f_{cm28} is the concrete mean compressive strength of 28 days in MPa; a (in days) and b are constant and t is the age of the concrete.

3. Basic assumption and material constitutive relationship

The hypotheses (essentially based on those introduced in initial studies of [6, 56, 59, 63, 64, 79, 90] in the elastic analysis of composite steel-concrete sections with stiff (rigid) shear connectors are assumed as following:

a) Bernoulli's concerning plane strain of cross-sections (Preservation of the plane cross section for the two elements considered compositely).
b) No vertical separation between parts, in other words identical vertical displacement at the slab-beam interface is assumed.
c) The connection system is distributed continuously along the axis of the beam.
d) The cross sections are free to deform (because they belong to statically determinate structures)

e) Concrete is not cracked $\sigma_c \leq (0.4 \div 0.5)\, R_c$.

f) For the service load analysis of these cross sections the stress levels are small and, therefore , linear elastic behavior may be assumed for the steel beam, in another words Hooke's law applies to steel as well as to concrete under short-time loads.

g) Moreover , for the concrete part, if the dependence of strains and stresses upon histories of water content and temperature is disregarded, with the exclusion of large strain reversals, and under normal environment conditions, the strain can be considered as a linear functional of the previous stress history alone. This linearity implies the principle of superposition [8, 9, 41, 42, 43, 80, 82, 83, 84, 92, 97], which states that strain response due to stress increments applied at different times may be added.

h) In the range of service ability loads concrete behaves in a way allowing to be treated as a linear viscoelastic body. On the basis of our assumptions for the purpose of structure analysis the total strain for concrete subjected to initial loading at time t_0 with a stress $\sigma(t_0)$ and subjected to subsequent stress variations $\Delta\sigma(t_i)$ at time t_i may be expressed as follows:

$$\varepsilon_{\mathrm{tot}}(t, t_0) - \varepsilon^{\mathrm{sh}}(t, t_0) = \sigma(t_0)\, J(t, t_0) + \int_{t_0}^{t} \frac{\mathrm{d}\sigma(\tau)}{\mathrm{d}\tau}\, J(t, \tau)\, \mathrm{d}\tau \ ,$$

where t is the time elapsed from casting of concrete; $\varepsilon_{\mathrm{tot}}(t, t_0)$ total axial strain; $\varepsilon^{\mathrm{sh}}(t, t_0)$ strain due to shrinkage, i.e. an elastic strain. Then the stress-strain behavior of concrete can be described with sufficient accuracy by the integral equations (1) by Bolztmann-Volterra [3, 21]

$$\varepsilon_c(t) = \frac{\sigma_c(t_0)}{E_c(t_0)}\, [1 + \phi(t, t_0)] + \int_{t_0}^{t} \frac{\mathrm{d}\sigma_c(\tau)}{\mathrm{d}\tau}\, \frac{1}{E_c(\tau)}\, [1 + \phi(t, \tau)]\, \mathrm{d}\tau$$

or according to ACI209R-92 we get

$$\begin{aligned} \varepsilon_c(t) = {} & \frac{\sigma_c(t_0)}{E_c(t_0)}\, [1 + 2.35\, \gamma_{c,RH}\, \gamma_{c,vs}\, \gamma_{c,s}\, \gamma_{c,\psi}\, \gamma_{sh,\alpha}\, \beta(t_0)\, \beta_c(t - t_0)] + \\ & + \int_{t_0}^{t} \frac{\mathrm{d}\sigma_c(\tau)}{\mathrm{d}\tau}\, \frac{1}{E_c(\tau)}\, [1 + 2.35\, \gamma_{c,RH}\, \gamma_{c,vs}\, \gamma_{c,s}\, \gamma_{c,\psi}\, \gamma_{sh,\alpha}\, \beta(\tau)\, \beta_c(t - \tau)]\, \mathrm{d}\tau \ , \end{aligned} \tag{16}$$

where $[2.35\, \gamma_{c,RH}\, \gamma_{c,vs}\, \gamma_{c,s}\, \gamma_{c,\psi}\, \gamma_{sh,\alpha}\, \beta(\tau)\, \beta_c(t - \tau)]$ is the so called the creep function, $\beta(\tau)$ depends on the age increase of concrete. It is called the function of aging, and it characterizes the process of the aging. The increase of τ makes $\beta(\tau)$ monotonously decrease. The function $\beta_c(t - \tau)$ (where t is the time interval during which the structure is under observation, τ is the running coordinate of time) characterizes the process of creeping. The constitutive law expressed by (10), represents the stress-strain-time relationship for the concrete slab.

i) The modulus of concrete elasticity is invariant in time t [38, 41, 92] i.e.

$$E_c(\tau) = E_c(t_0) = E_{\mathrm{const}} = E_{\mathrm{cm}t0} = 0.043\, \varrho_c^{1.5}\, \sqrt{f_{\mathrm{cm}t0}}\ (\mathrm{MPa}) \ ,$$

where ϱ_c is the weight of concrete (kg/m^3) and $f_{\mathrm{cm}t0}$ (MPa) is the mean concrete compressive strength at the time of loading. The general equation for prediction compressive strength at any time t is given by $f_{\mathrm{cm}t} = f_{\mathrm{cm}28}\, t/(a + b\,t)$, where $f_{\mathrm{cm}28}$ is the concrete mean compressive strength at 28 days in MPa; $a = 4$ in days, $b = 0.85$ is constants, t is the age of concrete.

j) According to a proposal by Sonntag [53], the influence of the development of the bending moment $M_{\mathrm{c,r}}(t)$ in the concrete member, upon the redistribution of the normal force of concrete $N_{\mathrm{c,r}}(t)$ can be neglected.

k) For the service load analysis no slip and uplift effects occurs between the steel and concrete.

l) A single theory of interaction ignoring shear lag effects is considered [66].

4. Basic equations of equilibrium

Let us denote both the normal forces and the bending moments in the cross-section of the plate and the girder after the loading in the time $t = 0$ with $N_{\mathrm{c},0}$, $M_{\mathrm{c},0}$, $N_{\mathrm{a},0}$, $M_{\mathrm{a},0}$ and with $N_{\mathrm{c,r}}(t)$, $M_{\mathrm{c,r}}(t)$, $N_{\mathrm{a,r}}(t)$, $M_{\mathrm{a,r}}(t)$ a new group of normal forces and bending moments, arising due to creep and shrinkage of concrete.

For a composite bridge girder with $J_{\mathrm{c}} = A_{\mathrm{c}}\,(n\,I_c)\,n/(A_{\mathrm{s}}\,I_{\mathrm{s}}) \leq 0.2$ according to the suggestion of Sonntag [94] we can write the equilibrium conditions in time t as follows

$$N(t) = 0\ , \qquad N_{\mathrm{c,r}}(t) = N_{\mathrm{a,r}}(t)\ , \tag{17}$$

$$\sum M(t) = 0\ , \qquad M_{\mathrm{c,r}}(t) + N_{\mathrm{c,r}}(t)\,r = M_{\mathrm{a,r}}(t)\ . \tag{18}$$

Due to the fact that the problem is a twice internally statically indeterminate system, the equilibrium equations (17), (18) are not sufficient to solve it.

It is necessary to produce two additional equations in the sense of compatibility of deformations of both steel girder and concrete slab in time t (Fig. 1).

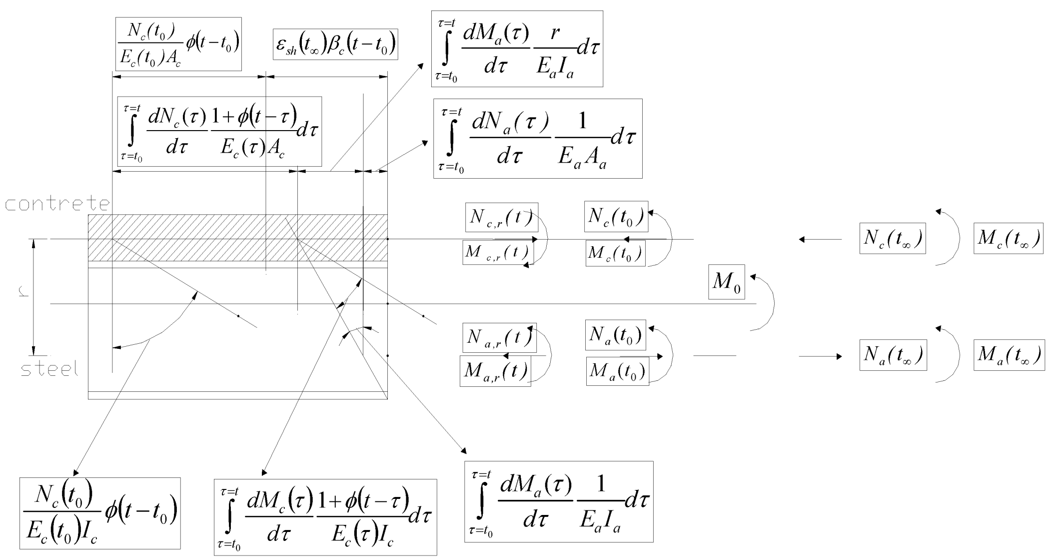

Fig.1: Mechano-mathematical model for deformations in cross-section in composite steel-concrete beam, regarding creep of the concrete

5. Deriving of the mechano-mathematical model

5.1. Strain compatibility on the contact surfaces between the concrete and steel members of composite girder

For constant elasticity module of concrete strain compatibility on the contact surfaces between the concrete and steel members of composite girder is as following:

$$\begin{gathered}\frac{N_{\mathrm{c},0}}{E_{\mathrm{c}}(t_0)\,A_{\mathrm{c}}}\,[1+2.35\,\gamma_{\mathrm{c}5}\,\beta(t_0)\,\beta_{\mathrm{c}}(t-t_0)]-\\ -\frac{1}{E_{\mathrm{c}}(t_0)\,A_{\mathrm{c}}}\int\limits_{t_0}^{t}\frac{\mathrm{d}N_{\mathrm{c,r}}(\tau)}{\mathrm{d}\tau}\,[1+2.35\,\gamma_{\mathrm{c}5}\,\beta(\tau)\,\beta_{\mathrm{c}}(t-\tau)]\,\mathrm{d}\tau+\\ +\frac{N_{\mathrm{a},0}}{E_{\mathrm{a}}\,A_{\mathrm{a}}}-\frac{1}{E_{\mathrm{a}}\,A_{\mathrm{a}}}\int\limits_{t_0}^{t}\frac{\mathrm{d}N_{\mathrm{a,r}}(\tau)}{\mathrm{d}\tau}=\frac{M_{\mathrm{a},0}}{E_{\mathrm{a}}\,I_{\mathrm{a}}}\,r+r\,\frac{1}{E_{\mathrm{a}}\,I_{\mathrm{a}}}\int\limits_{t_0}^{t}\frac{\mathrm{d}M_{\mathrm{a,r}}(\tau)}{\mathrm{d}\tau}\,\mathrm{d}\tau\;.\end{gathered}\tag{19}$$

Compatibility of Curvatures when $\tau = t$ is:

$$\begin{gathered}\frac{M_{\mathrm{c},0}}{E_{\mathrm{c}}(t_0)\,I_{\mathrm{c}}}\,[1+2.35\,\gamma_{\mathrm{c}5}\,\beta(t_0)\,\beta_{\mathrm{c}}(t-t_0)]-\\ -\frac{1}{E_{\mathrm{c}}(t_0)\,I_{\mathrm{c}}}\int\limits_{t_0}^{t}\frac{\mathrm{d}M_{\mathrm{c,r}}(\tau)}{\mathrm{d}\tau}\,[1+2.35\,\gamma_{\mathrm{c}5}\,\beta(\tau)\,\beta_{\mathrm{c}}(t-\tau)]\,\mathrm{d}\tau=\\ =\frac{M_{\mathrm{a},0}}{E_{\mathrm{a}}\,I_{\mathrm{a}}}+\frac{1}{E_{\mathrm{a}}\,I_{\mathrm{a}}}\int\limits_{t_0}^{t}\frac{\mathrm{d}M_{\mathrm{a,r}}(\tau)}{\mathrm{d}\tau}\,\mathrm{d}\tau\;.\end{gathered}\tag{20}$$

After integrating the two equations by parts and using the (17) and (18) for assessment of normal forces $N_{\mathrm{c,r}}(t)$ and bending moment $M_{\mathrm{c,r}}(t)$ two linear integral Volterra equations of the second kind are derived.

$$\begin{gathered}N_{\mathrm{c,r}}(t)=\lambda_{\mathrm{N}}\int\limits_{t_0}^{t}N_{\mathrm{c,r}}(\tau)\,\frac{\mathrm{d}}{\mathrm{d}\tau}[1+2.35\,\gamma_{\mathrm{c}5}\,\beta(\tau)\,\beta_{\mathrm{c}}(t-\tau)]\,\mathrm{d}\tau+\\ +\lambda_{\mathrm{N}}\,N_{\mathrm{c},0}\,2.35\,\gamma_{\mathrm{c}5}\,\beta(t_0)\,\beta_{\mathrm{c}}(t-t_0)\;,\end{gathered}\tag{21}$$

$$\begin{gathered}M_{\mathrm{c,r}}(t)=\lambda_{\mathrm{M}}\int\limits_{t_0}^{t}M_{\mathrm{c,r}}(\tau)\,\frac{\mathrm{d}}{\mathrm{d}\tau}[1+2.35\,\gamma_{\mathrm{c}5}\,\beta(\tau)\,\beta_{\mathrm{c}}(t-\tau)]\,\mathrm{d}\tau+\\ +\lambda_{\mathrm{M}}\,M_{\mathrm{c},0}\,2.35\,\gamma_{\mathrm{c}5}\,\beta(t_0)\,\beta_{\mathrm{c}}(t-t_0)-\lambda_{\mathrm{M}}\,\frac{E_{\mathrm{c}}\,I_{\mathrm{c}}}{E_{\mathrm{a}}\,I_{\mathrm{a}}}\,N_{\mathrm{c,r}}(t)\,r\;.\end{gathered}\tag{22}$$

in which

$$\lambda_{\mathrm{N}}=\left[1+\frac{E_{\mathrm{c}}\,A_{\mathrm{c}}}{E_{\mathrm{a}}\,A_{\mathrm{a}}}\left(1+\frac{A_{\mathrm{a}}\,r^2}{I_{\mathrm{a}}}\right)\right]^{-1}\;,\tag{23}$$

$$\lambda_{\mathrm{M}}=\left[1+\frac{E_{\mathrm{c}}\,I_{\mathrm{c}}}{E_{\mathrm{a}}\,I_{\mathrm{a}}}\right]^{-1}\;.\tag{24}$$

In each of these equations the functions:

$$N_{c,0}\, 2.35\, \gamma_{c5}\, \beta(t_0)\, \beta_c(t-t_0)\ , \qquad M_{c,0}\, 2.35\, \gamma_{c5}\, \beta(t_0)\, \beta_c(t-t_0)\ ,$$

$$\frac{d}{d\tau}[1 + 2.35\, \gamma_{c5}\, \beta(\tau)\, \beta_c(t-\tau)]\ , \qquad \gamma_{c5} = \gamma_{c,RH}\, \gamma_{c,vs}\, \gamma_{c,s}\, \gamma_{c,\psi}\, \gamma_{c,\alpha}$$

are given.

6. Numerical method

The integral equations (21, 22) are weakly singular Volterra integral equation of the second kind:

$$y(t) = g(t) + \lambda \int_{t_0}^{t} K(t,\tau)\, y(\tau)\, d\tau\ , \qquad t \in [t_0, T]\ , \qquad 0 < t_0 < T < \infty\ ,$$

where

$$g(t) = \lambda_N\, N_{c,0}\, 2.35\, \gamma_{c5}\, \beta(t_0)\, \beta_c(t-t_0)\ , \qquad \lambda = \lambda_N = \left[1 + \frac{E_c\, A_c}{E_a\, A_a}\left(1 + \frac{A_a\, r^2}{I_a}\right)\right]^{-1}$$

for (21) and

$$g(t) = \lambda_N\, N_{c,0}\, 2.35\, \gamma_{c5}\, \beta(t_0)\, \beta_c(t-t_0) - \lambda_M\, \frac{E_c\, I_c}{E_a\, I_a}\, N_{c,r}(t)\ , \qquad \lambda = \lambda_M = \left[1 + \frac{E_c\, I_c}{E_a\, I}\right]^{-1}$$

for (22) and

$$K(t,\tau) = \frac{d}{d\tau}\, [1 + 2.35\, \gamma_{c5}\, \beta(\tau)\, \beta_c(t-\tau)] = 2.35\, \gamma_{c5} \left[\beta_c(t-\tau)\, \frac{d\beta(\tau)}{d\tau} + \beta(\tau)\, \frac{d\beta_c(t-\tau)}{d\tau}\right]\ .$$

The singular kernel function can be written in the form:

$$K(t,\tau) = L(t,\tau)\, (t-\tau)^{-0.4}\ ,$$

where

$$L(t,\tau) = -1.25 \cdot 2.35\, \gamma_{c5} \left[\frac{0.118\, \tau^{-1.118}}{10 + (t-\tau)^{0.6}}\, (t-\tau) + \frac{6\, \tau^{-0.118}}{[10 + (t-\tau)^{0.6}]^2}\right]\ .$$

So in our case discontinuous kernel function $K(t,\tau)$ has an infinite singularity of type $(t-\tau)^{\gamma-1}$, $\gamma > 0$.

In order to solve (21, 22), we use the idea of product integration by considering the special case of:

$$y(t) = g(t) + \lambda \int_{t_0}^{t} L(t,\tau)\, (t-\tau)^{\gamma-1}\, y(\tau)\, d\tau\ , \quad t \in [t_0, T]\, ,\ 0 < t_0 < T < \infty\, ,\ 0 < \gamma < 1\ , \quad (25)$$

where the given functions $g(t)$ and $L(t,\tau)$ are sufficiently smooth which guarantee the existence and uniqueness of the solution (see Yosida, (1960), Miller & Feldstein, (1971)).

To solve (25) we use the method called product trapezoidal rule.

Let $n \geq 1$ be an integer and points $\{t_j = t_0 + j\,h\}_{j=0}^{n} \in [t_0, T]$. Then for general $y(t) \in C_{[t_0,T]}$ we define

$$(L(t,\tau)\,y(\tau))_n = \frac{1}{h}\left[(t_j-\tau)\,L(t,t_{j-1})\,y(t_{j-1}) + (\tau - t_{j-1})\,L(t,t_j)\,y(t_j)\right] \tag{26}$$

for $t_{j-1} \leq \tau \leq t_j$, $t \in [t_0, T]$.

This is piecewise linear in τ and it interpolates $L(t,\tau)\,y(\tau)$ at $\tau = t_0, \dots, t_n$. Using numerical approximation (26) we obtain the following method for solving the integral equation (25):

$$\tilde{y}_n(t_i) = g(t_i) + \lambda \sum_{j=0}^{i} \omega_{n,j}(t_i)\,[L(t_i,t_j)\,\tilde{y}_n(t_j)] \quad \text{for} \quad i = 0,1,\dots,n\ . \tag{27}$$

with weights

$$\omega_{n,0}(t_i) = \frac{1}{h}\int\limits_{t_0}^{t_1} (t_1-\tau)\,(t_i-\tau)^{\gamma-1}\,\mathrm{d}\tau\ ,$$

$$\omega_{n,n}(t_n) = \frac{1}{h}\int\limits_{t_{n-1}}^{t_n} (\tau-t_{n-1})\,(t_n-\tau)^{\gamma-1}\,\mathrm{d}\tau\ ,$$

$$\omega_{n,j}(t_i) = \frac{1}{h}\int\limits_{t_{j-1}}^{t_j} (\tau-t_{j-1})\,(t_i-\tau)^{\gamma-1}\,\mathrm{d}\tau + \frac{1}{h}\int\limits_{t_j}^{t_{j+1}} (t_{j+1}-\tau)\,(t_i-\tau)^{\gamma-1}\,\mathrm{d}\tau\ ,$$

for $i = 0,1,\dots,n$.

Calculating analytically the weights, we compute the approximate solution values $y_n(t_i)$ from the system (27).

Theorem 1. Consider the numerical approximation defined with piecewise linear interpolation (26). Then for all sufficiently large n, the equation (25) is uniquely solvable and moreover if $y(t) \in C^2_{[t_0,T]}$, then we have

$$||y - y_n|| \leq \frac{c\,h^2}{8} \max_{t_0 \leq t,\,\tau \leq T} \left| \frac{\partial^2 L(t,\tau)\,y(\tau)}{\partial \tau^2} \right| . \tag{28}$$

Since $L(t,.) \in C^2_{[t_0,T]}$, $t_0 \leq t \leq T$ the estimate (28) is immediate consequence of theorem 4.2.1 in Atkinson [4].

7. Numerical example

The method presented in the previous paragraph is now applied to a simply supported beam, subjected to a uniform load, whose cross section is shown in Fig. 2.

On the base of numerous solved examples the optimal step of one day for solving the integral equations (21, 22) is found. The elapsed time for solving the problem (27) for the period of twenty years (7300 days) is about up to ten minutes. For the period of forty years (14600 days) the elapsed time increases up to forty minutes.

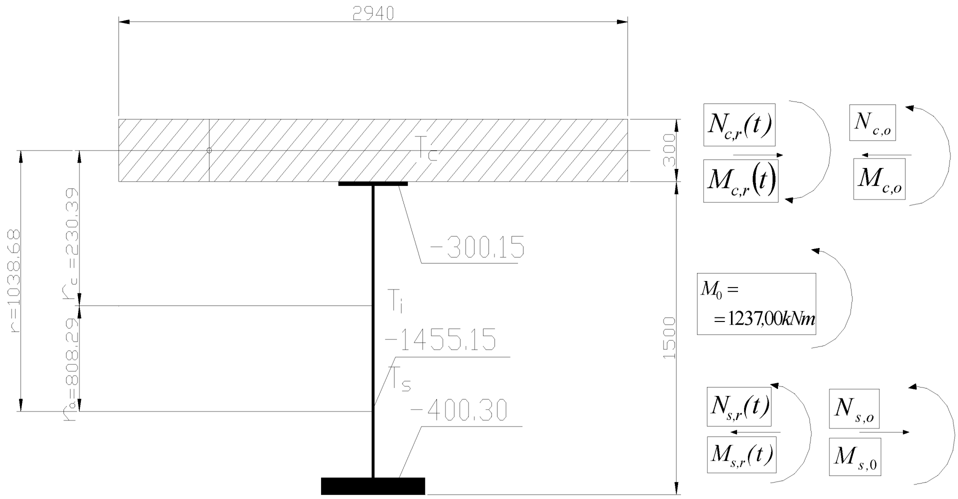

Fig.2: Composite beam with cross-section characteristic

$E_\mathrm{c} = 2.8178{\times}10^4\,\mathrm{MPa}$, $E_\mathrm{a} = 2.1{\times}10^5\,\mathrm{MPa}$, $A_\mathrm{c} = 8820\,\mathrm{cm}^2$, $A_\mathrm{a} = 383.25\,\mathrm{cm}^2$,
$n = \dfrac{E_\mathrm{a}}{E_\mathrm{c}} = 7.452$, $I_\mathrm{c} = 661500\,\mathrm{cm}^4$, $I_\mathrm{a} = 1207963.7\,\mathrm{cm}^4$, $r_\mathrm{c} = 25.407\,\mathrm{cm}$,
$r_\mathrm{a} = 78.463\,\mathrm{cm}$, $r = 103.870\,\mathrm{cm}$, $A_\mathrm{i} = 1566.8248\,\mathrm{cm}^2$, $I_\mathrm{i} = 4420140.76\,\mathrm{cm}^4$,
$M_0 = 1237\,\mathrm{kNm}$, $N_{\mathrm{c},0} = 837.286\,\mathrm{kN}$, $M_{\mathrm{c},0} = 24.716\,\mathrm{kNm}$, $M_{\mathrm{a},0} = 338.05\,\mathrm{kNm}$,

$$\lambda_\mathrm{N} = \left[1 + \frac{E_\mathrm{c}\,A_\mathrm{c}}{E_\mathrm{a}\,A_\mathrm{a}}\left(1 + \frac{A_\mathrm{a}\,r^2}{I_\mathrm{a}}\right)\right]^{-1} = 0.068220902\,, \quad \lambda_\mathrm{M} = \left[1 + \frac{E_\mathrm{c}\,I_\mathrm{c}}{E_\mathrm{a}\,I}\right]^{-1} = 0.931550028\,,$$

$RH = 08\,\%$ (humidity).

Mean 28-day strength: $f_\mathrm{cm28} = 33.3\,\mathrm{MPa}$,
($f_\mathrm{cm28} = 33.0\,\mathrm{MPa}$ according to CEB MC90-99),

Mean 28-day elastic modulus: $E_\mathrm{cm28} = 28178\,\mathrm{MPa}$,
($E_\mathrm{cm28} = 32009\,\mathrm{MPa}$ according to CEB MC90-99).

$\varrho_\mathrm{c} = 2345\,\mathrm{kg/m^3}$,

$$E_\mathrm{c}(\tau) = E_\mathrm{c}(t_0) = E_\mathrm{const} = E_{\mathrm{cm}t0} = 0.043 \cdot 2345^{1.5}\sqrt{33.30} = 28178\,\mathrm{MPa}$$
according to ACI 209R-92,

$\gamma_{\mathrm{c},t0} = 1.25\,t_0^{-0.118}$ corresponds to $\beta(t_0) = 0.61684$ for $t = 60$ days,
$\gamma_{\mathrm{c},RH} = 1.27 - 0.67\,h = 0.734$ for $h = 0.8$,

$$\gamma_\mathrm{c,vs} = \frac{2}{3}\left(1 + 1.13\,\mathrm{e}^{-0.0213(V/S)}\right) = 0.6975\,, \quad \text{where} \quad V/S = 150\,,$$

$\gamma_\mathrm{c,s} = 0.82 + 0.00624\,s = 1.018$, where $s = 75\,\mathrm{mm}$,
$\gamma_{\mathrm{c},\psi} = 0.88 + 0.0024\,\psi = 0.976$, where $\psi = 40$,
$\gamma_{\mathrm{c},\alpha} = 0.46 + 0.09\,\alpha \geq 1$ is air content factor, where $\alpha = 2$, $\gamma_{\mathrm{c},\alpha} = 1$,
$\beta_\mathrm{c}(36500 - 60) = 0.982004$.

8. Stress history analysis in midspan section of composite beam

In the concrete plate the normal component $N_\mathrm{c}(t_\infty) = N_\mathrm{c,0} - N_\mathrm{c,r}(t)$ and the bending moment $M_\mathrm{c}(t_\infty) = M_\mathrm{c,0} - M_\mathrm{c,r}(t)$ decrease by effect of creep (Figs. 3, 4). In the steel beam, the normal component $N_\mathrm{a}(t_\infty) = N_\mathrm{a,0} - N_\mathrm{a,r}(t)$ decreases and the bending moment $M_\mathrm{a}(t_\infty) = N_\mathrm{c,r}(t)\, r + M_\mathrm{c,r}(t)$ increases by the effect of creep (Fig. 5).

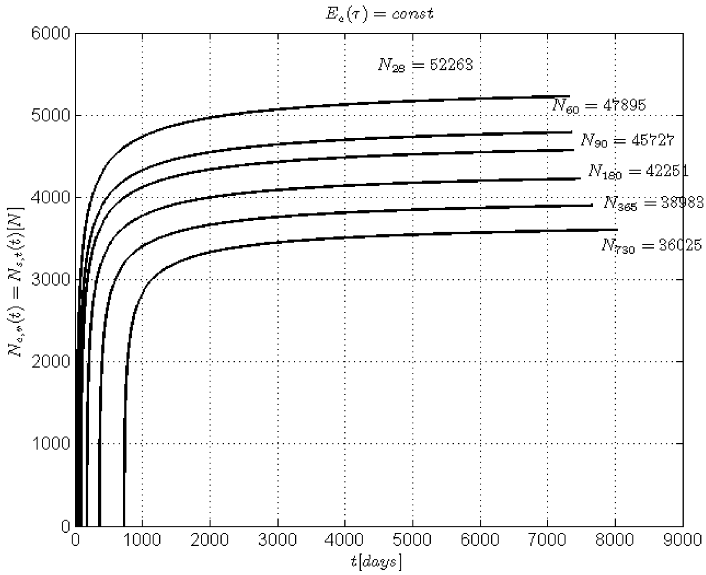

Fig.3: Values of normal forces $N_\mathrm{c,r}(t) = N_\mathrm{a,r}(t)$ in time t when loading is applied in time $t_0 = 28$, 60, 90, 180, 365 and 730 days (humidity 80 %)

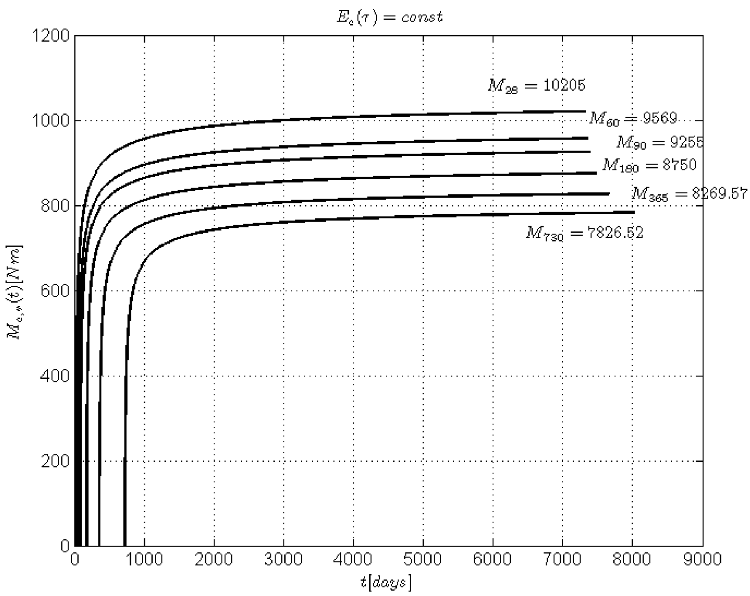

Fig.4: Values of bending moments $M_\mathrm{c,r}(t)$ in time t when loading is applied in time $t_0 = 28$, 60, 90, 180, 365 and 730 days (humidity 80 %)

The decrease of the stresses in concrete slab is accompanied by a gradual migration of stresses from the concrete slab to the steel beam. The decreasing of the stresses in the concrete slab is about 25 % from the initial values (Figs. 6, 7).

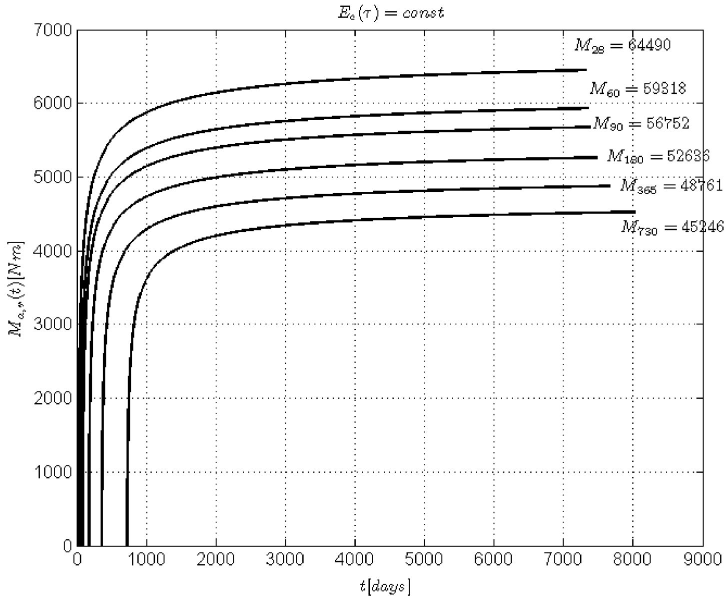

Fig.5: Values of bending moments $M_{\mathrm{a,r}}(t)$ in time t when loading is applied in time $t_0 = 28$, 60, 90, 180, 365 and 730 days (humidity 80 %)

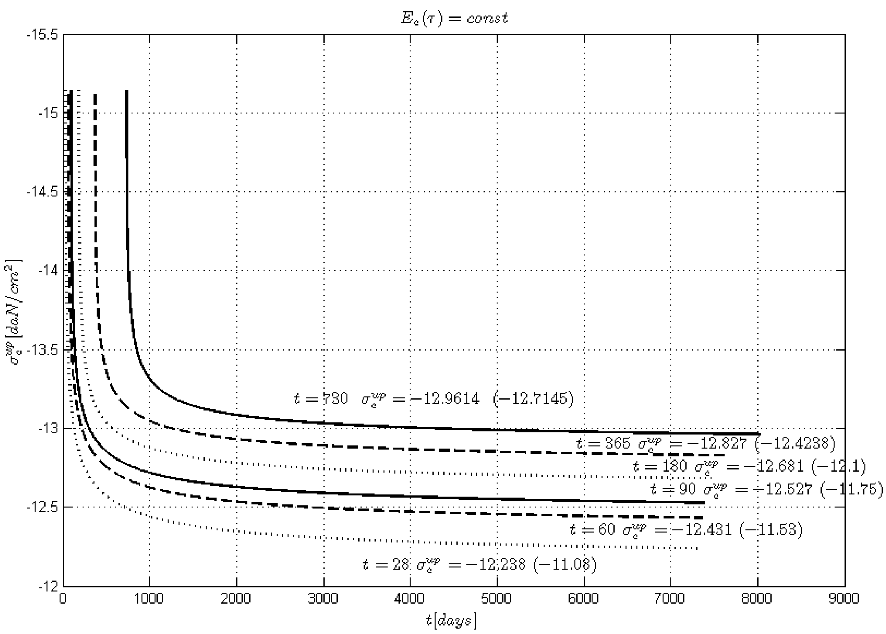

Fig.6: Values of normal stresses in upper fiber of concrete plate $\sigma_{\mathrm{c}}^{\mathrm{up}}(t)$ in time t_∞ when loading is applied in time $t_0 = 28$, 60, 90, 180, 365 and 730 days (humidity 80 %)

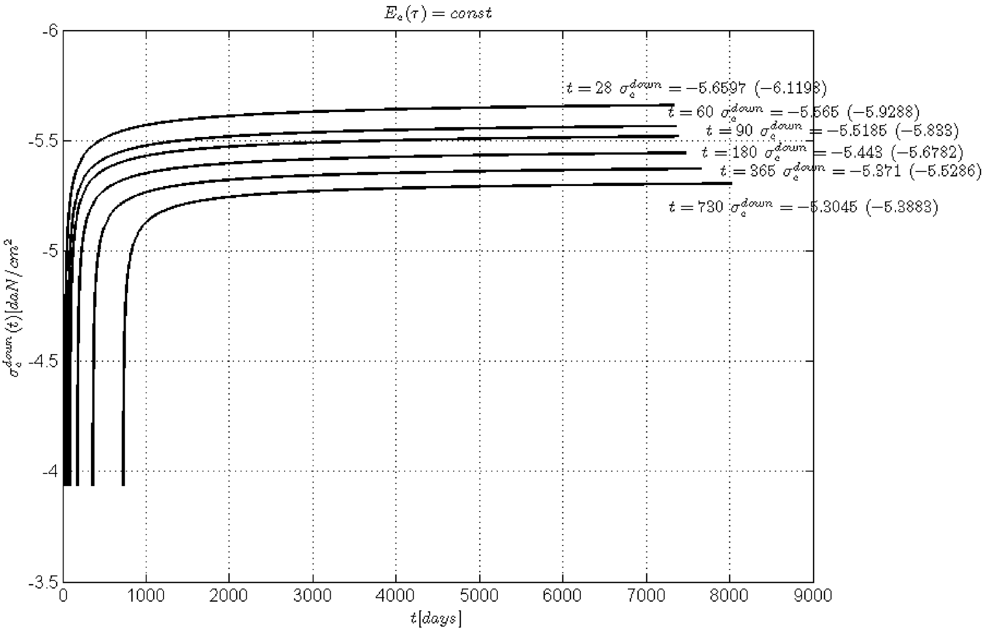

Fig.7: Values of normal stresses in down fiber of concrete plate $\sigma_c^{\mathrm{down}}(t)$ in time t_∞ when loading is applied in time t_0 = 28, 60, 90, 180, 365 and 730 days (humidity 80 %)

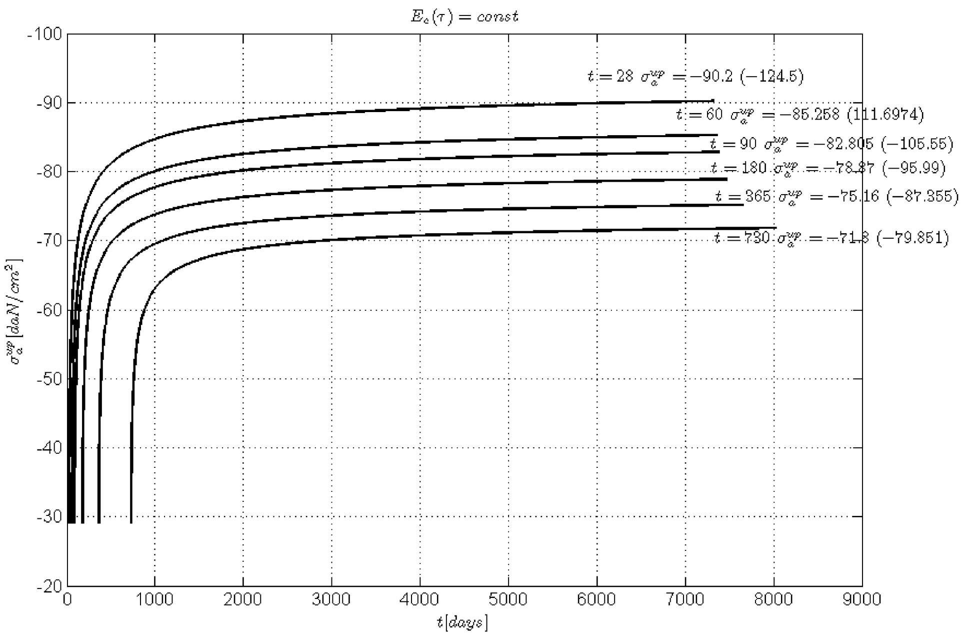

Fig.8: Values of normal stresses in upper fiber of steel girder $\sigma_a^{\mathrm{up}}(t)$ in time t_∞ when loading is applied in time t_0 = 28, 60, 90, 180, 365 and 730 days (humidity 80 %)

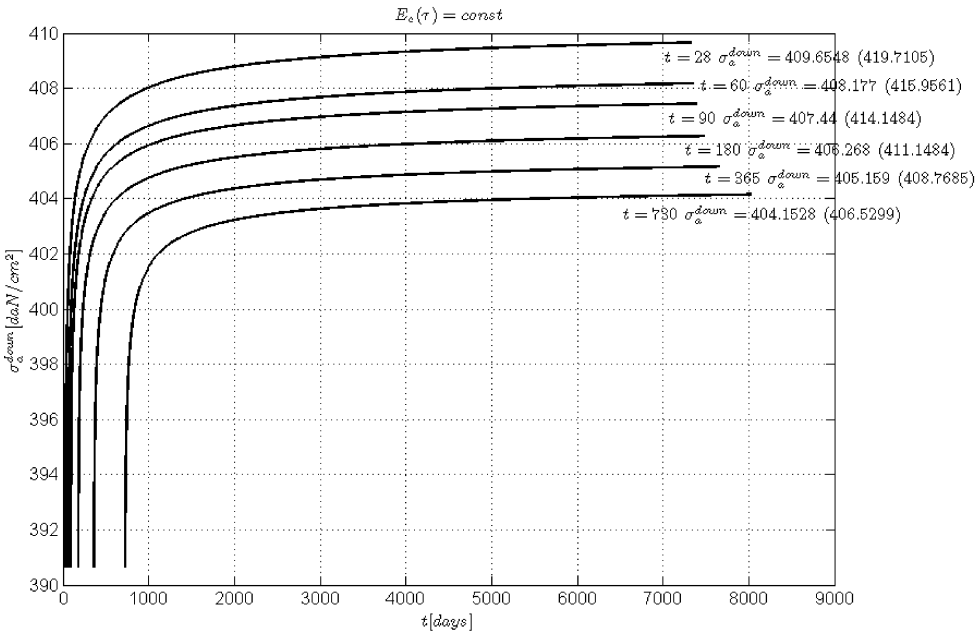

Fig.9: Values of normal stresses in down fiber of steel girder $\sigma_a^{\rm down}(t)$ in time t_∞ when loading is applied in time $t_0 = 28$, 60, 90, 180, 365 and 730 days (humidity 80 %)

Forces, moments, stresses	$RH = 90\,\%$	$RH = 80\,\%$	$RH = 70\,\%$	$RH = 60\,\%$	$RH = 50\,\%$
$M_{\rm c,r}$ (Nm)	9052.1	9569	10049	10491	10902
$M_{\rm a,r}$ (Nm)	54475	59318	64091	68779	73396
$N_{\rm c,r}$ (N)	43730	47895	52029	56116	60165
$\sigma_{\rm c}^{\rm up}$ (MPa)	−1.25961	−1.2431	−1.2276	−1.21294	−1.199
$\sigma_{\rm c}^{\rm down}$ (MPa)	−0.5495	−0.5565	−0.56269	−0.56809	−0.57283
$\sigma_{\rm a}^{\rm up}$ (MPa)	−8.06051	−8.5258	−8.9849	−9.43643	−9.88173
$\sigma_{\rm a}^{\rm down}$ (MPa)	40.68127	40.8177	40.95138	41.08196	41.20995

Tab.1: Values of normal forces, bending moments and normal stresses in time t_∞ when loading is applied in time $t_0 = 60$ for different humidity RH

The analysis of the obtained results show a very strong increase in the upper flange (Fig. 8), which final values are two to three times higher than the initial values and small increase (less than 6 % of the initial stress) of the stress in the bottom flange (Fig. 9). Figure 8 shows how the stress at the top fibers of the steel section undergoes strong increases in time.

Consequently, the stress history in the top flange of the steel beam becomes the most interesting aspect of this study.

These graphs also show how important is the age of concrete at loading. The later we impose the load on composite beam, the less is the influence of the concrete creep on the time behavior of the beam.

According to the proposed numerical method, we can conclude that the stresses in the top flange of the steel beam, for low values of parameter $t_0 = 28$ days and $t_0 = 60$ days, increase more for young concrete and less for old one for $t_0 = 365$ days and $t_0 = 730$ days.

Above all the influence of concrete age at loading time t_0 is significant only when its values are very low (i.e. with young concrete).

For the five standard cases assumed by ACI209R-92: $RH = 50\,\%$ corresponds to dry conditions (inside) and $RH = 90\,\%$ corresponds to humid conditions (outside) is made analysis for stress level in composite beams, performance in table 1.

9. Time development of deflections according to the received numerical results

When the distribution of the bending moments in steel section $M_{\mathrm{a}}(t_\infty) = M_{\mathrm{c,r}}(t) + N_{\mathrm{c,r}}(t)\, r$ is known, it is possible to calculate the change of the vertical deflections in time t. The Figure 10 shows the values of deflection in midspan section of composite beams in time t_∞. As it can be observed the change of the initial time when the loading moment M_0 is applied, has very considerable influence in the time development of deflections.

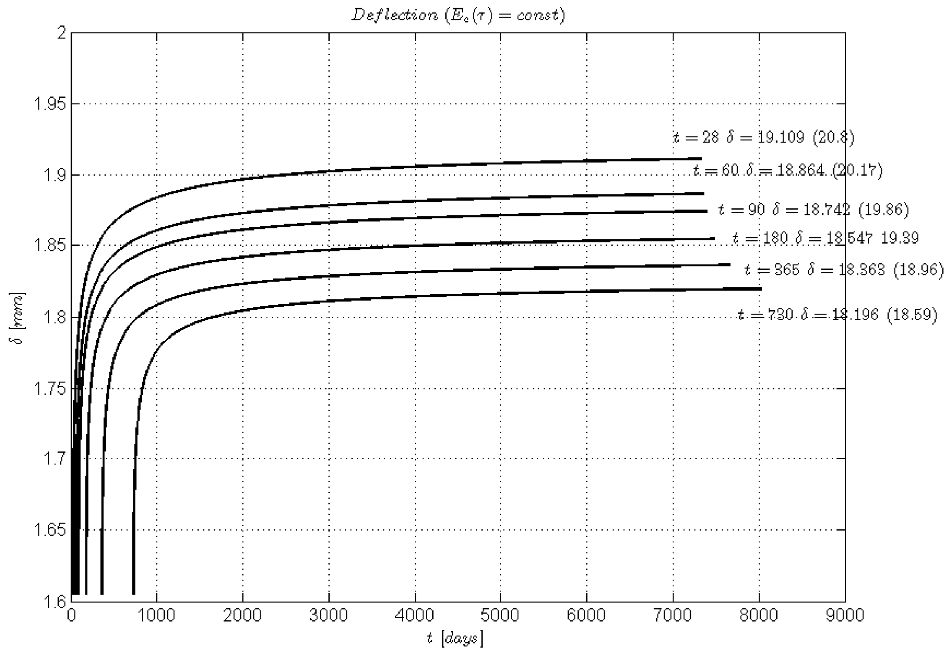

Fig.10: Values of deflection of steel girder (composite steel-concrete) delta(t) in time t_∞ when loading is applied in time t_0 = 28, 60, 90, 180, 365 and 730 days (humidity 80 %)

In practice the deflection in time t_∞ is determined by the following formulae:

$$\delta(t_\infty) = \frac{5}{48}\,\frac{M_0\,L^2}{E_{\mathrm{a}}\,I_{\mathrm{i,y}}} = \frac{5 \cdot 1237{\times}10^6 \cdot 34000^2}{48 \cdot 210000 \cdot 37.641838420{\times}10^9} = 18.844\,\mathrm{mm}\ .$$

According to the described above numerical method we get the following formulae for calculating the deflection. If the moment M_0 and the inertia moment $I_{\mathrm{i,y}}$ are replaced with $M_{\mathrm{a}}(t_\infty)$ and I_{a} respectively we get:

$$\delta(t_\infty) = \frac{5}{48}\,\frac{M_{\mathrm{a}}(t_\infty)\,L^2}{E_{\mathrm{a}}\,I_{\mathrm{a}}} = \frac{5 \cdot (338.05 + 59.2863){\times}10^6 \cdot 34000^2}{48 \cdot 210000 \cdot 12.079{\times}10^9} = 18.861\,\mathrm{mm}\ .$$

In every case considered above the elastic deflection $\delta(t)$ in time t_0 is the same we receive from the formulas:

$$\delta(t_0=0)=\frac{5}{48}\,\frac{M_0\,L^2}{E_{\mathrm{a}}\,I_{\mathrm{i,y}}(t_0)}=\frac{5\cdot 1237\times 10^6\cdot 34000^2}{48\cdot 210000\cdot 44.2014076\times 10^9}=16.047\,\mathrm{mm}\ ,$$

$$\delta(t_0=0)=\frac{5}{48}\,\frac{M_{\mathrm{a},0}\,L^2}{E_{\mathrm{a}}\,I_{\mathrm{a}}}=\frac{5\cdot(338.05)\times 10^6\cdot 34000^2}{48\cdot 210000\cdot 12.079\times 10^9}=16.047\,\mathrm{mm}\ .$$

according to our proposal.

10. Comparision with effective modulus methods (EMM)

This method uses the Dischinger's idea for applying in the calculation the ideal (fictitious) modulus of elasticity [36, 37, 49–51, 54, 55, 101]:

$$E_{\mathrm{ci}}=\frac{E_{\mathrm{cm}}}{1+\psi_{\mathrm{L}}\,\phi_{\mathrm{t}}}=\frac{E_{\mathrm{cm}}}{1+1.1\,\phi_{\mathrm{t}}}\ ,$$

where ϕ_{t} is a final creep coefficient of concrete.

It is applied to solve practical case shown in figure 2. The results obtained by manual method according ACI209R-92 model in comparison with CEB MC90-99 are illustrated in tables 2 and 3.

Type of beams	Characteristic	Steel	Composite (in $t_0=0$)	Composite (in $t=\infty$)	Dimensions
Height	h_i	1500	1800	1800	mm
Area	A_i	38325	156682	92635	mm^2
Static moment to down surface	S_{y0}	23428688	218728072	115052670	mm^3
Gravity center	e_{top}	888.7	404	558	mm
Gravity center	e_{bottom}	611.3	1396	1242	mm
Moment of inertia	$I_{i,\mathrm{y}}$	12079015497	44201407600	37641838420	mm^4
Section modulus	$W_{i,\mathrm{y,ct}}$	—	109409425	−67458492	mm^3
Section modulus	$W_{i,\mathrm{y,cb}}$	—	425013535	−145898598	mm^3
Section modulus	$W_{i,\mathrm{y,at}}$	−13592026	−425013535	−145898598	mm^3
Section modulus	$W_{i,\mathrm{y,ab}}$	19759036	31662899	30307438	mm^3

Tab.2: Dimensions of steel and composite beams

Stress in time t_0	$t_0=60$ days	Unit	Stress in time t_∞	$t_\infty=36500$ days
M_0	1237	kNm	M_0	1237
$n_0=E_{\mathrm{a}}/E_{\mathrm{cm}}$	7.452 (6.36)	—	$n_{\mathrm{L}}=n_0\,(1+\psi_{\mathrm{L}}\,\phi_{\mathrm{t}})$, $\psi_{\mathrm{L}}=1.1$	14.87 (18.62)
$\sigma_{\mathrm{c}}^{\mathrm{top}}=M/W_{i,\mathrm{y,ct}}/n_0$	−1.517 (−1.600)	MPa	$\sigma_{\mathrm{c}}^{\mathrm{top}}=M/W_{i,\mathrm{y,ct}}/n_{\mathrm{L}}$	−1.233 (−1.200)
$\sigma_{\mathrm{c}}^{\mathrm{bottom}}=M/W_{i,\mathrm{y,cb}}/n_0$	−0.390 (−0.300)	MPa	$\sigma_{\mathrm{c}}^{\mathrm{bottom}}=M/W_{i,\mathrm{y,cb}}/n_{\mathrm{L}}$	−0.57 (−0.60)
$\sigma_{\mathrm{a}}^{\mathrm{top}}=M/W_{i,\mathrm{y,at}}$	−2.91 (−2.20)	MPa	$\sigma_{\mathrm{a}}^{\mathrm{top}}=M/W_{i,\mathrm{y,at}}$	−8.47 (−11.00)
$\sigma_{\mathrm{a}}^{\mathrm{bottom}}=M/W_{i,\mathrm{y,ab}}$	39.06 (38.80)	MPa	$\sigma_{\mathrm{a}}^{\mathrm{bottom}}=M/W_{i,\mathrm{y,ab}}$	40.81 (41.50)

Tab.3: Level of stresses of composite beams according ACI209R-92 model, in comparison with CEB MC90-99

11. Conclusion

A numerical method for time-dependent analysis of composite steel-concrete sections according ACI 209R-92 model is presented. Using MATLAB code a numerical algorithm was developed and subsequently applied to a simple supported beam. These numerical procedures, suited to a PC, are employed to better understand the influence of the creep of the concrete in time-dependent behavior of composite section.

For the service load analysis, this method makes it possible to follow with great precision the migration of the stresses from the concrete slab to the steel beam, which occurs gradually during the time as a result of the creep of the concrete. At the same time, it is possible to calculate the deflections in the half-span section according to ACI 209R-92. Both these effects have a considerable importance in time-dependent response of composite beams.

The results obtained by this numerical method according to the ACI 209R-92 provision are completely comparable with the results based on effective modulus method (EMM) proposed by EUROCODE 4. The values in the figures 6–10 shown in brackets are obtained by numerical method according to CEB MC90-99 [78]. They differ from the corresponding results of ACI 209R-92 slightly from practical point of view.

The numerical aging linear viscoelastic solution presented in § 6 and 7 allows some interesting consideration on the general trends of the time dependent effects on composite beams, that may be useful in the preliminary and conceptual design stages of all structures of this type.

A gradual migration of stresses from the concrete slab to the steel beam is observed as a consequence of the decrease of the stresses in concrete slab. The decreasing of the stresses in the concrete slab in upper fibers is about 25 % from the initial values according to ENV 1992-1-1 and ACI 209R-92 (Fig. 6). The values of migration stresses in the concrete slab depend on the age of the concrete at loading time t_0.

It results in a very strong increase in the upper flange (Fig. 8) and small increase of the stress in the bottom flange (less than 8 % of the initial stress, Fig. 9). Figure 8 shows how the stress at the top fibers of the steel section undergoes strong increases in time: the final values are four to six times higher than the initial values. The stress in the steel part of the composite beam increases more for young concrete and little for old one.

The relative humidity causes considerable variations to the final stress (see table 1) in comparison with CEB MC90-99. The lower is the value of humidity, the higher is the stresses in the steel beam.

According to our results based on numerous practical examples we can state that about 90–92 % of the maximum values of the stressed in concrete or steel in time t_∞ are reached after about three years. Besides that 98 % are reached after about twenty years in comparison with the period of hundred years obtained by the EM Method [51, 52].

In our opinion the influence of creep on time dependent behavior of composite steel-concrete beams according to ACI 209R-92 code provisions, in comparison with CEB FIB model code-1990 is underestimated. It is observed from the numerical results shown on figures 3–10.

Finally, the creep effect must be carefully evaluated in order to fully understand the behavior of the structure. The numerical methods proposed in this paper can used to control the deflection in every test in composite beams sustained at service loads during the time t. It means that we can proof the regulars of the theory of the concrete creep.

The most important conclusion of our investigation is that considering the creep effect, using the fundamental integral equations (16) of the aging linear viscoelasticity problem, a universal numerical method has been elaborated for statically determinate bridge composite plate girder according to the ACI 209R-92 model. This method allows the use of a perfect linear theory of concrete creep i.e. the theory of the viscoelastic body of Boltzman-Volttera-Maslov-Arutyunyan-Trost-Bazant.

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The article is dedicated to the scientific haritage of the great italian mathematician Vito Volterra (1860–1949).